Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **October/November 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1(i)	State or imply ordinates 0.915929…, 1, 1.112485…	**B1**
	Use correct formula, or equivalent, with h = 1.2 and three ordinates	**M1**
	Obtain answer 2.42 only	**A1**
		3
1(ii)	Justify the given statement	**B1**
		1

Question	Answer	Marks
2	Use law for the logarithm of a power or a quotient on the given equation	**M1**
	Use $\log_2 8 = 3$ or $2^3 = 8$	**M1**
	Obtain $x^2 - 8x - 8 = 0$, or horizontal equivalent	**A1**
	Solve a 3-term quadratic equation	**M1**
	Obtain final answer x = 8.90 only	**A1**
		5

Question	Answer	Marks
3	Use correct $\tan(A \pm B)$ formula and express LHS in terms of tan θ	**M1**
	Using tan 60° = $\sqrt{3}$ and $\cot\theta = 1/\tan\theta$, obtain a correct equation in $\tan\theta$ in any form	**A1**
	Reduce the equation to one in $\tan^2\theta$ only	**M1**
	Obtain $11\tan^2\theta = 1$, or equivalent	**A1**
	Obtain answer 16.8°	**A1**
		5

Question	Answer	Marks
4(i)	Use correct product or quotient rule or rewrite as $2\sec x - \tan x$ and differentiate	M1
	Obtain correct derivative in any form	A1
	Equate the derivative to zero and solve for x	M1
	Obtain $x = \frac{1}{6}\pi$	A1
	Obtain $y = \sqrt{3}$	A1
		5
4(ii)	Carry out an appropriate method for determining the nature of a stationary point	M1
	Show the point is a minimum point with no errors seen	A1
		2

Question	Answer	Marks
5	Separate variables and obtain $\int \frac{1}{y} \mathrm{d}y = \int \frac{x+2}{x+1} \mathrm{d}x$	B1
	Obtain term $\ln y$	B1
	Use an appropriate method to integrate $(x+2)/(x+1)$	*M1
	Obtain integral $x + \ln(x+1)$, or equivalent, e.g. $\ln(x+1) + x + 1$	A1
	Use $x = 1$ and $y = 2$ to evaluate a constant, or as limits	DM1
	Obtain correct solution in x and y in any form e.g. $\ln y = x + \ln(x+1) - 1$	A1
	Obtain answer $y = (x+1)\mathrm{e}^{x-1}$	A1
		7

Question	Answer	Marks
6(i)	State or imply $3x^2y + x^3\frac{dy}{dx}$ as derivative of x^3y	**B1**
	State or imply $9xy^2\frac{dy}{dx} + 3y^3$ as derivative of $3xy^3$	**B1**
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	**M1**
	Obtain the given answer AG	**A1**
		4
6(ii)	Equate numerator to zero and use $x = -y$ to obtain an equation in x or in y	**M1**
	Obtain answer $x = a$ and $y = -a$	**A1**
	Obtain answer $x = -a$ and $y = a$	**A1**
	Consider and reject $y = 0$ and $x = y$ as possibilities	**B1**
		4

Question	Answer	Marks
7(i)	State modulus 2	**B1**
	State argument $-\frac{1}{3}\pi$ or $-60°$ ($\frac{5}{3}\pi$ or $300°$)	**B1**
		2
7(ii)	*EITHER:* Expand $(1-(\sqrt{3})\text{i})^3$ completely and process i^2 and i^3	**(M1**
	Verify that the given relation is satisfied	**A1)**
	OR: $u^3 = 2^3\left(\cos(-\pi) + i\sin(-\pi)\right)$ or equivalent: follow their answers to **(i)**	**(M1**
	Verify that the given relation is satisfied	**A1)**
		2

Question	Answer	Marks
7(iii)	Show a circle with centre $1-(\sqrt{3})\mathrm{i}$ in a relatively correct position	**B1**
	Show a circle with radius 2 passing through the origin	**B1**
	Show the line Re $z = 2$	**B1**
	Shade the correct region	**B1**
		4

Question	Answer	Marks
8(i)	State or imply the form $\dfrac{A}{1-x}+\dfrac{B}{2x+3}+\dfrac{C}{(2x+3)^2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 1$, $B = -2$, $C = 5$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
	[Mark the form $\dfrac{A}{1-x}+\dfrac{Dx+E}{(2x+3)^2}$, where $A = 1$, $D = -4$, $E = -1$, **B1M1A1A1A1** as above.]	
8(ii)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(1+\frac{2}{3}x)^{-1}$, $(2x+3)^{-1}$, $(1+\frac{2}{3}x)^{-2}$ or $(2x+3)^{-2}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A3 FT**
	Obtain final answer $\frac{8}{9}+\frac{19}{27}x+\frac{13}{9}x^2$, or equivalent	**A1**
		5

Question	Answer	Marks
9(i)	Integrate by parts and reach $ax^{\frac{3}{2}}\ln x + b\int x^{\frac{3}{2}}.\frac{1}{x}\,dx$	*M1
	Obtain $\frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{2}{3}\int x^{\frac{1}{2}}\,dx$	A1
	Obtain integral $\frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}}$, or equivalent	A1
	Substitute limits correctly and equate to 2	DM1
	Obtain the given answer correctly AG	A1
		5
9(ii)	Evaluate a relevant expression or pair of expressions at $x = 2$ and $x = 4$	M1
	Complete the argument correctly with correct calculated values	A1
		2
9(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 3.031	A1
	Show sufficient iterations to 5 d.p. to justify 3.031 to 3 d.p., or show there is a sign change in the interval (3.0305, 3.0315)	A1
		3

Question	Answer	Marks
10(i)	State or imply a correct normal vector to either plane, e.g. $\mathbf{i}+\mathbf{j}+3\mathbf{k}$ or $2\mathbf{i}-2\mathbf{j}+\mathbf{k}$	B1
	Carry out correct process for evaluating the scalar product of two normal vectors	M1
	Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result	M1
	Obtain final answer 72.5° or 1.26 radians	A1
		4
10(ii)	*EITHER*: Substitute $y = 2$ in both plane equations and solve for x or for z	(M1
	Obtain $x = 3$ and $z = 1$	A1)
	OR: Find the equation of the line of intersection of the planes	
	Substitute $y = 2$ in line equation and solve for x or for z	(M1
	Obtain $x = 3$ and $z = 1$	A1)

Question	Answer	Marks
	EITHER: Use scalar product to obtain an equation in a, b and c, e.g. $a + b + 3c = 0$	**(B1**
	Form a second relevant equation, e.g. $2a - 2b + c = 0$, and solve for one ratio, e.g. $a : b$	***M1**
	Obtain final answer $a : b : c = 7 : 5 : -4$	**A1**
	Use coordinates of A and values of a, b and c in general equation and find d	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
	*OR*1: Calculate the vector product of relevant vectors, e.g. $(\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \times (2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	**(*M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $7\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$	**A1**
	Substitute coordinates of A in plane equation with their normal and find d	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
	*OR*2: Using relevant vectors, form a two-parameter equation for the plane	**(*M1**
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	**A1 FT**
	State 3 correct equations in x, y, z, λ and μ	**A1 FT**
	Eliminate λ and μ	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
	*OR*3: Use the direction vector of the line of intersection of the two planes as normal vector to the plane	**(*M1**
	Two correct components	**A1**
	Three correct components	**A1**
	Substitute coordinates of A in plane equation with their normal and find d	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
		7